MICROWAVE FILTERS WITH SINGLE ATTENUATION POLES AT REAL OR IMAGINARY FREQUENCIES

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Abstract

A new unified theory is presented for the synthesis of exactly equi-ripple low pass prototypes having (a) one simple pole of attenuation at a real frequency, or (b) a single pair of real axis transmission zeros (giving linear phase performance). Practical results for microwave bandpass filters are described.

Introduction

This paper describes two classes of microwave filters which have quite distinct applications, yet are closely related mathematically and in physical realization. The filters are derived from a low pass prototype which is optimally equi-ripple in the pass band. The first type has a pair of single-ordered transmission zeros on the $j\omega$ axis, and the second has a pair of single-ordered transmission zeros on the σ axis of the complex $(\sigma + j\omega)$ plane. The transmission zeros may be realized by cross coupling a pair of non-adjacent elements in the filter, negatively for the first type and positively for the second. The first type of filter gives improved skirt attenuation performance and the second improved pass band delay compared with the ordinary Chebyshev filter.

The first application of coupling between non-adjacent resonators at microwave frequencies appears to have originated with Kurzrok[1,2]. He showed that to obtain finite frequency attenuation poles it was necessary to reverse the "natural" phase of the extra cross coupling. It was not until much later that Rhodes[3] showed that when the cross couplings have the same phase as the direct couplings then the finite transmission zeros produced are either complex or on the real axis of the complex frequency plane. Hence they may be used to design filters having non-minimum-phase characteristics, e.g. linear phase filters[4]. Other authors have employed cross couplings to realize elliptic function filters, and recently more general types having both finite attenuation poles and delay equalization have been described[5,6].

Filters with several cross couplings tend to be relatively difficult to tune, encouraging some designers to look at the possibility of synthesizing high-ordered filters having just one or two finite frequency transmission zeros[7,8]. However, the low pass filters discussed in Reference (7) are not equi-ripple and are therefore non-optimum. The band pass filters described by Cristal[8] are for a somewhat limited application (broad-band interdigital filters).

The New Prototype Filters

(a) Real frequency attenuation poles.

It is perhaps surprising that the low pass rational function to be introduced apparently has not been utilized previously, at least for the applications described here, since it is derived by a straightforward application of Chebyshev's theorem, a brief description of which may be found in Reference (9). The rational function has simple poles at $x = \pm a$, and takes the form

$$f(x) = \frac{(a+\sqrt{a^2-1})^2 x T_{n-1}(x) - 2a^2 T_{n-2}(x) + (a-\sqrt{a^2-1})^2 x T_{n-3}(x)}{2(a^2-x^2)} \quad ..(1)$$

It is shown schematically in Fig. 1 for the case n = 8 with real value for the parameter a. The low pass prototype filter has an insertion loss given by

$$L = 1 + h^2 f^2(x) \quad .... (2)$$

where h is small for small pass band ripple levels, and the maximum pass band return loss is defined as

$$A_R = 10 \log_{10}(1+1/h^2) \text{ dB} \quad .... (3)$$

The location of the stop band minimum shown in Fig. 1 is given by x_2, where

$$x_2^2 = a^2 + 2a\sqrt{a^2-1}/(n-2) \quad .... (4)$$

and the filter attenuation at this minimum is

$$A_s = 10 \log_{10}[1+h^2 f^2(x_2)] \text{ dB} \quad .... (5)$$

Since $1/h^2 \gg 1$ and $h^2 f^2(x_2) \gg 1$, we have

$$A_R + A_S = 20 \log_{10} f(x_2) \text{ dB} \quad .... (6)$$

invariant of the pass band ripple level.

The edge of the stop band is the value x_1 indicated in Fig. 1, and is derived numerically by iteration. This enables the value of $A_R + A_S$ given by (6) to be plotted as a function of x_1, as shown in Fig. 2 for filters of degree 3 through 10. Similar graphs are available for Chebyshev and optimum elliptic function filters[10], and when the characteristics of the three types of filters are compared it is found that they are almost exactly parallel to one another for any given degree, e.g. for degree 8 the Chebyshev filter plot of $A_R + A_S$ parallels that shown in Fig. 2 at ordinate values 22 dB lower, while the elliptic filter parallels it at ordinate values 18 dB higher. Note that the single pole filters of degree 3 and 4 are identical to the elliptic filter, as expected. The improvement in skirt attenuation over a Chebyshev filter upon introducing finite poles is quite large when one finite pole is introduced, and becomes successively less as subsequent poles are introduced. Evidently the extra complexity of the elliptic function filter may not always be justified.

Exact synthesis of the element values of the low pass prototypes may be carried out , but an approximate technique described later is found to give acceptable results.

(b) Filters with real axis transmission zeros (Linear Phase filters).

If we take the function (1)and make the substitution $a = j\sigma$ where σ is real, then

$$f(x) = \frac{(\sqrt{\sigma^2+1}+\sigma)^2 x T_{n-1}(x) - 2\sigma^2 T_{n-2}(x) + (\sqrt{\sigma^2+1}-\sigma)^2 x T_{n-3}(x)}{2(\sigma^2+x^2)} \quad .. (7)$$

Hence the insertion loss function for an equi-ripple low pass filter having a pair of real axis transmission zeros is given by (2) with f(x) represented by (7). Again, this may be synthesized exactly , but the following approximation suffices for most applications.

Approximate synthesis of single pole filters.

This may be achieved by introducing cross coupling between one pair of non-adjacent elements of the standard Chebyshev low pass prototype filter. The latter is shown in admittance inverter form[4] for n even in Fig. 3 (the theory for the case n odd is similar). The element values are given by the well known formulas

$$g_1 = \frac{2\sin\frac{\pi}{2n}}{\gamma}$$

$$g_r g_{r-1} = \frac{4\sin\frac{(2r-1)\pi}{2n}\sin\frac{(2r+1)\pi}{2n}}{\gamma^2+\sin^2\frac{r\pi}{n}}$$

$$(r = 1,2,....m) \qquad m = n/2$$

$$\gamma = \sinh\left(\frac{1}{n}\sinh^{-1}\frac{1}{h}\right)$$

$$S = (\sqrt{1+h^2}+h)^2 \text{ (the pass-band VSWR).}$$

$$J_m = \sqrt{S} \text{ .... m odd or } 1/\sqrt{S} \text{ .... m even}$$

$$J_{m-1} = 0 \text{ .... for Chebyshev filters.} \quad .. (8)$$

Normally for the Chebyshev filter the extra cross coupling admittance inverter J_{m-1} is not present. In order to introduce the attenuation poles it can be shown that the value of J_{m-1} required is given by

$$J_{m-1} = \frac{-J_m}{(ag_m)^2 - J_m^2} \quad .... (9)$$

This formula holds for both negative and positive cross coupling, (substitute $a=j\sigma$ in (8) to obtain the latter). In order to maintain a good VSWR at mid band it is necessary to change the value of J_m slightly according to the formula

$$J_m' = \frac{J_m}{1+J_m J_{m-1}} \quad .... (10)$$

Analysis shows that in practice the VSWR is well maintained over the entire band. In practical filter realizations an exact equi-ripple response is achievable by fine tuning.

Experimental results

The first realizations of microwave band pass filters having finite real frequency attenuation poles were described by Kurzrok[1,2]. Other realizations using waveguide cavities[5] or combline[6] are also possible. However , currently the majority of applications are for filters having real axis transmission zeros, which may be located at approximately $\sigma=\pm1$ to give a very convenient form of linear phase filter. A more generalized category of linear phase filters has been described by Rhodes[4], but in general these require several extra cross couplings and are somewhat more difficult to construct and tune. The use of only one extra positive cross coupling limits the extent of the delay compensation to about 50% of the passband, but this is exactly the requirement for many communications systems.

The mechanical construction of an 8-cavity waveguide filter in WR137 is shown in Fig.4, and its performance,compared with that of a 7-cavity Chebyshev filter having the same bandwidth and ripple level, is shown in Fig.5. The improved amplitude and delay of the self-equalized filter are obtained at the cost of slightly increased insertion loss and the loss of "one cavity" of attenuation compared with the Chebyshev filter.

Conclusions

The results presented demonstrate an interesting unification of the theories for the two cases of real frequency or real axis transmission zeros. The approximate synthesis given is sufficiently precise for most practical applications, as demonstrated by the results which have been obtained.

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References

1. R.M.Kurzrok,"General three-resonator filters in waveguide", IEEE Trans. on Microwave Theory and Techniques,Vol.MTT-14, pp.46-47, January 1966.
2. R.M.Kurzrok,"General four-resonator filters at microwave frequencies" , IEEE Trans.on Microwave Theory and Techniques, Vol.MTT-14, pp.295-296, June 1966.
3. J.D.Rhodes,"The theory of generalized interdigital networks", IEEE Trans. on Circuit Theory, Vol.CT-16, pp.280-288, August 1969.

4. J.D.Rhodes,"A low pass prototype network for microwave linear phase filters", "The generalized interdigital linear phase filter", "The generalized direct-coupled cavity linear phase filter",IEEE Trans.on Microwave Theory and Techniques,Vol.MTT-18, pp.290-313, June 1970.

5. A.E.Atia and A.E.Williams,"Nonminimum-phase optimum amplitude bandpass waveguide filters" , IEEE Trans.on Microwave Theory and Techniques, Vol. MTT-22, pp.425-431 , April 1974.

6. R.Levy,"Mixed lumped and distributed linear phase filters" , 1974 European Conference on Circuit Theory and Design, IEE(London) Conference Publication No.116, pp.32-37.

7. M.C.Agarwal and A.S.Sedra,"On designing sharp cut-off low pass filters",IEEE Trans on Audio and Electroacoustics, Vol.AU-20, pp.138-141, June 1972.

8. E.G.Cristal,"Design equations for a class of wide-band bandpass filters" , IEEE Trans.on Microwave Theory and Techniques, Vol.MTT-20,pp.696-699, October 1972.

9. R.Levy,"Generalized rational function approximation in finite intervals using Zolotarev functions", IEEE Trans. on Microwave Theory and Techniques, Vol.MTT-18, pp.1052-1064, December 1970.

10. R.Saal,"Der Entwurf von filtern mit hilfe des Kataloges normierter tiefpasse",Telefunken A.G.,Backnang(Wurttenberg), West Germany, 1966, pp.7,8.

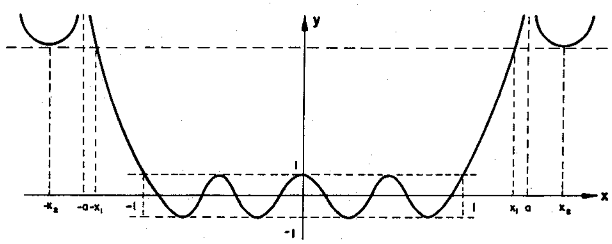

FIG 1. RATIONAL FUNCTION WITH SINGLE POLE (CASE n = 8)

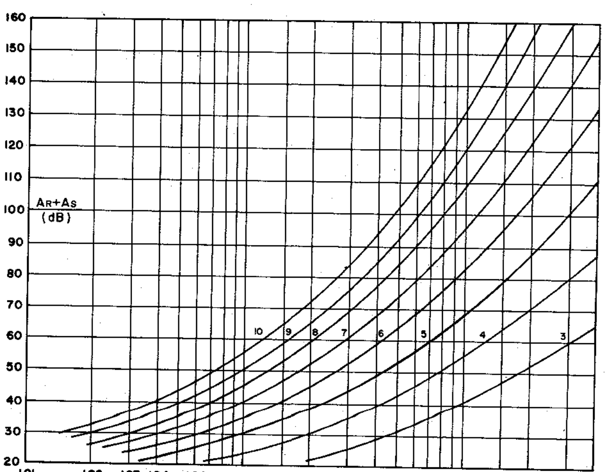

FIG 2. UNIVERSAL CHARACTERISTICS FOR SINGLE-POLE FILTERS

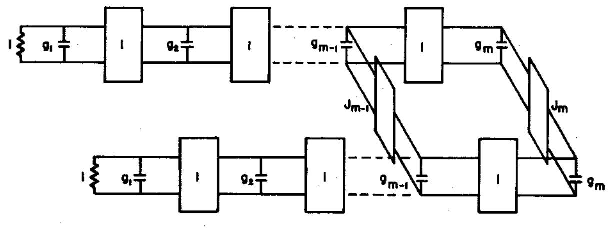

Fig.3. LOW PASS PROTOTYPE FILTER.

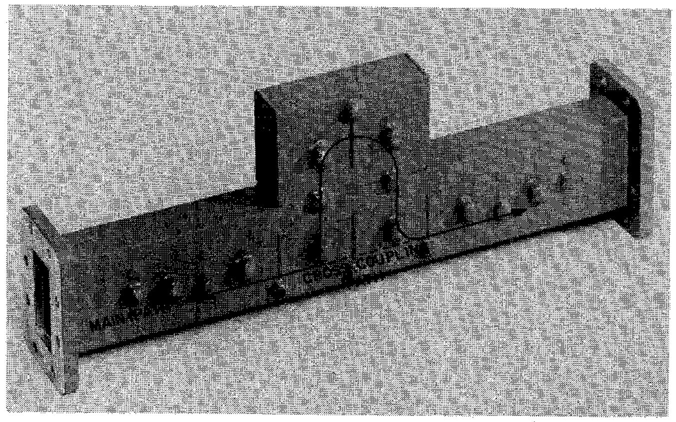

FIG 4. 8-CAVITY WR 137 WAVEGUIDE LINEAR PHASE FILTER

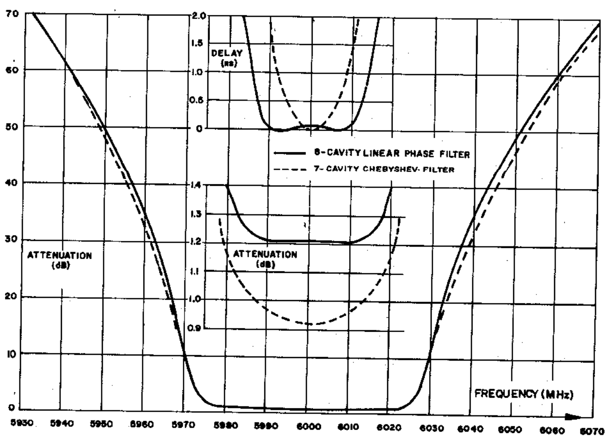

FIG 5. COMPARISON OF 8-CAVITY LINEAR PHASE FILTER WITH 7-CAVITY CHEBYSHEV FILTER